Finite Element Analysis of Local Flux Density Variation Considering PWM Current Harmonics

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This paper presents the method of analyzing flux density changes in the iron core as permanent magnet synchronous motor is operated by PWM sine wave. Large amount of harmonics are contained in the current in case sinusoidal current is created by PWM where a significant amount of time for analysis is required. This paper suggests a method of calculating the change in flux density of each element in a short time with non-linear and linear FEA based on two dimensional magneto static FEA.

***Index Terms*— electric motor, finite element analysis, magnetic cores, magnetic flux density**

I. Introduction

The electric motors are usually designed to produce smooth torque when the current waveform conforms to a particular shape[1]. The PWM inverters are widely used to supply the particular current wave form. Closed-loop current-regulated PWM inverters can supply current waveforms close to the ideal current waveform, but there are large amount of higher current harmonics is mixed in the current. And to do reflect such current harmonics in the motor analysis, location of the rotor must be changed at multiple times, more than twice the number of the harmonic number, to conduct non-linear FEA.

Since such analysis requires too much time, this research will discuss a method to analyze PWM current in a short time, using non-linear FEA and linear FEA concurrently.

II. Method of Analysis

Electric current provided to motor from PWM inverter includes higher current harmonics, which can be depicted as (1).

$$\begin{aligned} i(t) &= I_1 \sin(\omega t - \phi_1) + I_2 \sin(2\omega t - \phi_2) \\ &+ I_3 \sin(3\omega t - \phi_3) + I_4 \sin(4\omega t - \phi_4) + \cdots \end{aligned} \quad (1)$$

In the method suggested by this research, the analysis process can be largely divided by two stages. The first stage is non-linear FEA using only the fundamental harmonic, and the second stage is linear FEA using all current harmonics. In the second stage, each element is linearized using the analysis results obtained in the first stage.

A. Stage I: Non-linear FEA with Fundamental current wave

As shown in (2), the fundamental harmonic includes only the first term of (1).

$$i_{\text{fund}}(i) = I_1 \sin(\omega t - \phi_1) \quad (2)$$

In this stage, non-linear FEA is conducted using (3)[1], while changing rotor location in certain intervals under current input as in (2).

$$\nabla \times \frac{1}{\mu}(\nabla \times \mathbf{A}) = \mathbf{J} + \nabla \times \left(\frac{\mu_0}{\mu} \mathbf{M} \right) \quad (3)$$

Where μ is permeability, **A** is magnetic vector potential, **J** is current density and **M** is magnetization. In Stage I, **M** is always 0 in the element at iron core.

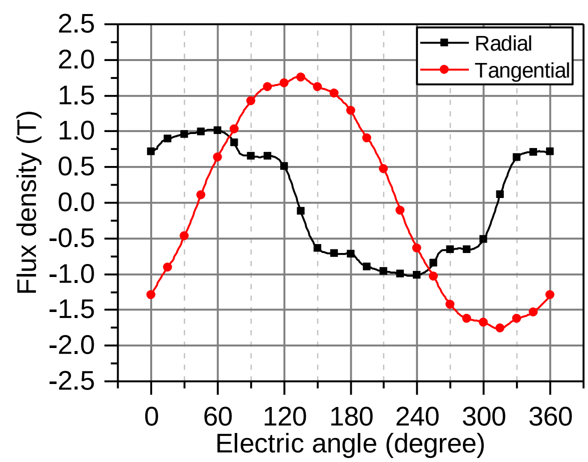

Fig. 1. Flux density variation in an element

As the result of Stage I, flux density vector values sampled at regular intervals can be obtained in all elements, as shown in Fig. 1. From these values, values in between the intervals can be predicted using methods such as spline interpolation or Fourier approximation, and the predicted values are used for linearizing each element in the second stage.

B. Stage II: Linear FEA with Harmonic Current Wave

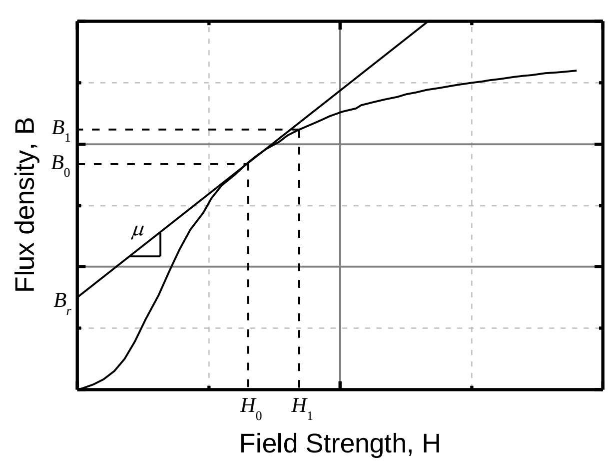

Fig. 2. B-H curve and differential permeability

After the analysis in Stage I, the result can be used to linearize each element.

Let's suppose a magnetic circuit, consisting iron core and coils. The iron core is made from a material that has single-valued B-H curve as in Fig. 2. Let's put the case that a random location within the core has field strength $\mathbf{H}_0$ and flux density $\mathbf{B}_0$ while currents I_0 are flowing in coils. In this situation, when the size of the currents increases by ΔI, field strength and flux density is also increased by $\Delta\mathbf{H}$ and $\Delta\mathbf{B}$ respectively. If ΔI is small enough, the relationship between field strength **H** and flux density **B** can be assumed to be linear during the time when the current changes from I_0 to $I_0+\Delta I$.

In equation (1), the higher harmonics I_2, I_3, I_4… are significantly smaller than fundamental harmonic I_1; so such relationship explained above is applicable. In Stage II, each element's flux density obtained from Stage I is interpolated to linearize each element. In the analysis result obtained in Stage I, if an element has field strength and flux density with direction θ and the size of B_0 and H_0 at a certain time, the element can be linearized with a material that has differential permeability and residual flux density as below [1].

$$\mu_d = \left.\frac{\partial B}{\partial H}\right|_{H=H_0}, \qquad B_r = B_0 - \mu_d H_0 \tag{4}$$

The relationship in above formula is shown in Fig. 2. Differential permeability μ_d can be expressed as the tangent line slope of B-H curve at a point when $H=H_0$, and residual flux density B_r can be expressed as the point where an extended line from the tangent line meets B axis.

Each element's flux density for PWM current can be obtained by conducting linear FEA with current described in (1) in a model linearized as described above.

III. Results and Discussion

For currents consisting higher harmonics, Existing non-linear FEA method and results obtained from the suggested method are displayed in Fig. 3, as example.

As can be seen from the enlarged graph on the bottom-left side in Fig. 3, result of suggested method is almost approaching the Non-linear FEA, and computing time was approximately 1:15 as shown in TABLE I

More detailed analysis result will be provided and compared in a full paper.

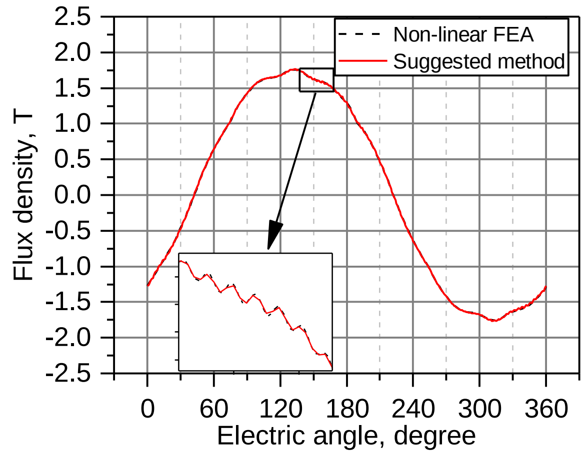

Fig. 3. Flux density variation in an element with PWM current

TABLE I
Computing Times

Method	Computing time
Non-linear FEA at every step	700 Minutes
proposed method	45 Minutes

References

[1] J. R. Hendershot & T. J. E. Miller, *Design of brushless permanent-magnet machines,* Motor Design Books LLC, 2010, pp. 370

[2] M. V. K. Chari and S.J. salon, "Numerical methods in electromagnetism," Academic press, 2000, pp. 338

[3] N. Demerdash, F. Fouad and T. Nehl, "Determination of winding inductances in ferrite type permanent magnet electric machinery by finite elements," in IEEE Transactions on Magnetics, vol. 18, no. 6, pp. 1052-1054, Nov 1982.

[4] M. Gyimesi and D. Ostergaard, "Inductance computation by incremental finite element analysis," IEEE Transactions on Magnetics, vol. 35, no. 3, pp. 1119-1122, May 1999.

[5] T. W. Nehl, F. A. Fouad and N. A. Demerdash, "Determination of Saturated Values of Rotating Machinery Incremental and Apparent Inductances by an Energy Perturbation Method," in IEEE Transactions on PAS, vol. PAS-101, no. 12, pp. 4441-4451, Dec. 1982.